1.1 Real Number System:

Rational & Irrational Numbers

Standard:
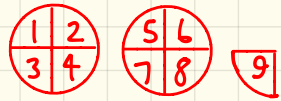
N.RN.3

- Integers — $1, 2, 3, \ldots$ • Fractions — $\dfrac{1}{2}, \dfrac{5}{7}, \dfrac{2}{3}$

- Improper Fractions — $\dfrac{10}{7}, \dfrac{9}{2}, \dfrac{4}{3}$

- Mixed Numbers — $5\dfrac{1}{2}, 2\dfrac{3}{8}, 10\dfrac{5}{6}$
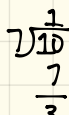
Let's move in between the different forms:

[Improper Fraction to Mixed Number]   Use Mental Long Division

① $\dfrac{9}{4} = 2\dfrac{1}{4} \cdots$ ⟨Visual⟩
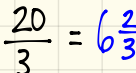
$4\overline{)9} \;\; \begin{array}{c} 2 \\ \underline{8} \\ 1 \end{array}$

- There are 9 fourths.
- There are 2 wholes and 1 fourth.

② $\dfrac{10}{7} = 1\dfrac{3}{7}$

$7\overline{)10} \;\; \begin{array}{c} 1 \\ \underline{7} \\ 3 \end{array}$

③ $\dfrac{20}{3} = 6\dfrac{2}{3}$

[Mixed Number to Improper Fraction]   Multiply Whole Number with Fraction's Denominator, Add the result to the Fraction's Numerator
— The improper fraction's denominator will be the same as the mixed number's denominator

① $2\dfrac{1}{4} = \dfrac{9}{4}$ ② $1\dfrac{3}{7} = \dfrac{10}{7}$ ③ $6\dfrac{2}{3} = \dfrac{20}{3}$

$2 \times 4 + 1 = 9$ $1 \times 7 + 3 = 10$ $6 \times 3 + 2 = 20$

[Fraction to Decimal] Use Long Division

① $\dfrac{1}{3}$ ② $\dfrac{9}{4}$ ③ $\dfrac{2}{5}$ ④ $\dfrac{4}{7}$

①
```
     .33
  3)1.00
    9
   ──
    10
```
Repeating Decimal!

②
```
     2.25
  4)9.00
    8
   ──
    10
    8
   ──
    20
```
Terminating Decimal

③
```
     .4
  5)2.0
    20
   ──
    0.
```
Terminating Decimal

④
```
     .5714...
  7)4.00
    35
   ──
    50
    49
   ──
    10
    7
   ──
    30
```
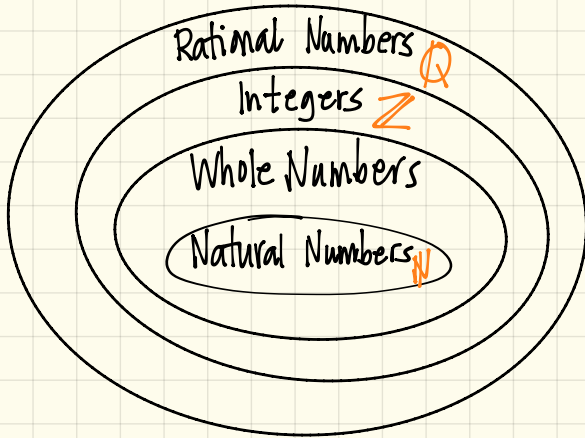Repeating Decimal
(decimal too long to see
repeating pattern)

[new] The Real Number System

· <u>Natural Numbers</u> — 1, 2, 3, 4, ... (positive whole numbers)

· <u>Whole Numbers</u> — 0, 1, 2, 3, 4, ...   (natural numbers plus zero)

· <u>Integers</u> — ...-3, -2, -1, 0, 1, 2, 3, ... (positive & natural numbers plus zero)

· <u>Rational Numbers</u> — any quotient fraction, $\dfrac{p}{q}$, where p & q are integers.
  — $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{1}{5}, \dfrac{1}{1}, ...$ (repeating & terminating decimals)

· <u>Irrational Numbers</u> — repeating decimals (decimal that does not terminate)
  — π , e , $\sqrt{2}$
  (3.14...) (2.71...)

Real Numbers Systems $\mathbb{R}$

Rational Numbers $\mathbb{Q}$

Integers $\mathbb{Z}$

Whole Numbers

Natural Numbers $\mathbb{N}$

Irrational Numbers $\mathbb{Q}$

- Involving Decimals that repeat or terminate
- Any decimal that can be made into a fraction

- Involving Decimals that do not terminate
- Any decimal that can not be made into a fraction.

[Example] Classify the type of Real Number.

① 5 — Natural, Whole, Integer, Rational

② 0.575 — Rational Number

③ $\sqrt{7}$ — Irrational Number

④ $5.\overline{75}$ — Rational Number

⑤ $\dfrac{\sqrt{10}}{2}$ — Irrational Number

⑥ $\sqrt{9}$ — Natural, Whole, Integer, Rational

Let's consider the combination of rational and irrational numbers by operations:

[Case 1] Let's discuss the sum of rational numbers

(Examples)

① $5 + 6 = 11$

integer → so rational

② $6 + \dfrac{1}{4} = \dfrac{24}{4} + \dfrac{1}{4} = \dfrac{25}{4}$

fraction → so rational

(conclusion) Sum of Rational Numbers will have a result of a Rational Number.

[Case 2] Let's discuss the product of rational numbers

(Examples)

① $(5)(6) = 30$

Integer → so rational

② $(6) \cdot \left(\dfrac{1}{4}\right) = \dfrac{6}{4}$

fraction → so rational

(conclusion) Product of Rational Numbers will have a result of a Rational Number.

[Case 3] Let's discuss the sum of a rational & irrational number

(Examples)

① $5 + \pi$
$= 5 + 3.14...$
$= 8.14...$

can't be fraction → irrational

② $7 + e$
$= 7 + 2.71...$
$= 9.71...$

can't be fraction → irrational

(conclusion) Sum of Rational & Irrational Number will have a result of a Irrational Number.

Case 4 Let's discuss the product of a rational & irrational number

(Examples)

① $5 \cdot \sqrt{7}$
   $= 5\sqrt{7}$
   $\approx 13.228\ldots$
   ↗
   Can't be fraction → irrational

② $7 \cdot \pi$
   $= 7\pi$
   $= 21.991\ldots$
   ↗
   Can't be fraction → irrational

Conclusion Product of Rational & Irrational Number will have a result of a <u>Irrational Number</u>

Rule of Thumb ⟶ Irrational Numbers cotaminate everything! Especially when you add & multiply

note: This only works with • rationals to rationals
                                    • irrationals to rationals.

Results of the combination between irrational to irrational may result in either irrational or rational.