2.7 Solving System of Equations Algebra using Substitution & Elimination

Standards:

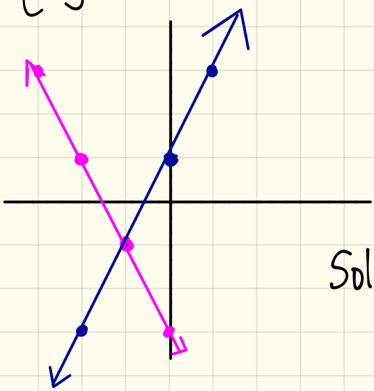
A.REI.5

A.REI.11

$\boxed{\text{Old}}$ Solving systems of Equations by graphing

Find the solution.

1. $\begin{cases} y = 2x + 1 \\ y = -2x - 3 \end{cases}$



Check Answer: $(-1,-1)$

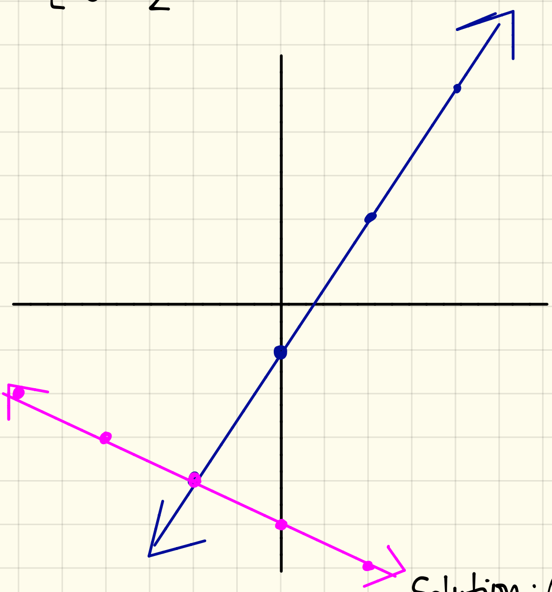
$x = -1, y = -1$

$-1 = 2(-1) + 1$
$-1 = -2 + 1$
$-1 = -1$ ✓

$-1 = -2(-1) - 3$
$-1 = 2 - 3$
$-1 = -1$ ✓

Solution: $(-1,-1)$

2. $\begin{cases} y = \frac{3}{2}x - 1 \\ y = \frac{-x}{2} - 5 \end{cases}$



Check Answer: $(-2,-4)$ $x=-2, y=-4$

$-4 = \frac{3}{2}(-2) - 1$
$-4 = -3 - 1$
$-4 = -4$ ✓

$-4 = \frac{-(-2)}{2} - 5$
$-4 = 1 - 5$
$-4 = -4$ ✓

Solution: $(-2,-4)$

(Conclusion) When solving systems of equations by graphing <u>both</u> equations need to be in slope-intercept form.

$$\begin{cases} y = mx+b \\ y = mx+b \end{cases}$$

$\boxed{\text{new-A}}$ Solving Systems of Equations Algebraically (using substitution)

Let's consider $\begin{cases} -2x+y=1 \\ y=2x-3 \end{cases}$.

We know that we can solve the system of equations (graphically) by converting 1^{st} equations into slope-intercept form.

Can we solve the system algebraically? (without graphing)

What property can we use to help use to solve the system of equations (algebraically)? $\Longrightarrow$ $\boxed{\text{Transitive Property}}$

If $a=b$, $b=c$ then $a=c$.

$\begin{cases} -2x+y=1 \\ y=-2x-3 \end{cases}$ $\longrightarrow$ Equation is solved for y.
Can we use the transitive property? yes!

$$\begin{cases} -2x + y = 1 \\ y = -2x - 3 \end{cases}$$

$$\Longrightarrow -2x + (-2x-3) = 1$$
$$-2x - 2x - 3 = 1$$
$$-4x - 3 = 1$$
$$-4x \cancel{-3+3} = 1+3$$
$$\frac{\cancel{-4}x}{\cancel{-4}} = \frac{4}{-4}$$
$$x = -1$$

Sub -1 for x:

$$y = -2x - 3$$
$$y = -2(-1) - 3$$
$$y = 2 - 3$$
$$y = -1.$$

Solution: $(-1, -1)$.

Conclusion: To solve systems of equation by substitution, at least 1 equation needs to be solved for a variable.

[Example 1] Solve.

$$\begin{cases} x - 2y = 4 \\ x = y + 2 \end{cases}$$

$$\Longrightarrow (y+2) - 2y = 4$$
$$y + 2 - 2y = 4$$
$$-y + 2 = 4$$
$$-y \cancel{+2-2} = 4 - 2$$
$$\frac{\cancel{-}y}{\cancel{-1}} = \frac{2}{-1}$$
$$y = -2.$$

Sub -2 for y:

$$x = y + 2$$
$$x = -2 + 2$$
$$x = 0.$$

Solution: $(0, -2)$.

[Example 2] Solve.

$$\begin{cases} y = 6x \\ y = 5x + 7 \end{cases}$$

$$\Longrightarrow 6x = 5x + 7$$
$$6x - 5x = \cancel{5x - 5x} + 7$$
$$x = 7$$

sub 7 for x:

$$y = 6(7)$$
$$y = 42.$$

Solution: (7, 42).

[Example 3] Solve.

$$\begin{cases} 3x - 4y = 0 \\ x = -y \end{cases}$$

$$\Longrightarrow 3(-y) - 4y = 0$$
$$-3y - 4y = 0$$
$$-7y = 0$$
$$y = 0.$$

Sub 0 for y:

$$x = -y$$
$$x = -(0)$$
$$x = 0.$$

Solution: (0, 0).

$\boxed{\text{new-B}}$ Solving Systems of Equations Algebraically (using elimination)

Let's consider $\begin{cases} -2x+y=1 \\ 2x+y=-3. \end{cases}$

We know how to solve this:

1) by graphing — converting both equations to slope-intercept
2) by substitution – solving for a variable in at least 1 equation.

Can we solve this system any other way easier?

What property can we use to help use to "eliminate" a variable? $\Longrightarrow$ $\boxed{\text{Additive Inverse Property}}$

$$a+(-a)=0.$$

$\begin{cases} -2x+y=1 \\ 2x+y=-3 \end{cases}$ $\longrightarrow$ Looking at both equations, do you see additive inverses? yes!

$$\begin{array}{r} \cancel{-2x}+y=1 \\ \cancel{2x}+y=-3 \\ \hline 2y=-2 \\ y=-1. \end{array}$$

Sub -1 for y:

$$2x+(-1)=-3$$
$$2x-1=3$$
$$2x\cancel{-1+1}=3+1$$
$$\frac{\cancel{2}x}{\cancel{2}}=\frac{4}{2}$$
$$x=2.$$

Solution $(2,-1)$.

Conclusion To solve system of equations by elimination, both equations in standard form and eliminate one variable by adding the additive inverses.

[Example 1] Solve by elimination.

$$\begin{cases} 2x - 3y = 12 \\ 4x + 3y = 24 \end{cases}$$

$$\begin{array}{r} 2x - \cancel{3y} = 12 \\ 4x + \cancel{3y} = 24 \\ \hline 6x = 36 \\ x = 6. \end{array}$$

Sub 6 in for x:

$$4(6) + 3y = 24$$
$$24 + 3y = 24$$
$$\cancel{24 - 24} + 3y = 24 - 24$$
$$3y = 0$$
$$y = 0.$$

Solution: $(6, 0)$.

[Example 2] Solve.

$$\begin{cases} -4x + y = 8 \\ -3x + 4y = -7 \end{cases}$$

$$-4(-4x + y = 8)$$
$$-3x + 4y = -7$$

$$\Rightarrow \begin{array}{r} 16x - \cancel{4y} = -32 \\ -3x + \cancel{4y} = -7 \\ \hline 13x = -39 \\ x = -3 \end{array}$$

Sub -3 for x:

$$-4(-3) + y = 8$$
$$12 + y = 8$$
$$12 - 12 + y = 8 - 12$$
$$y = -4$$

Solution: $(-3, -4)$

[Example 3] Solve.

$$\begin{cases} -3x+7y=-16 \\ -9x+5y=16 \end{cases}$$

$$-3(-3x+7y=-16) \qquad\qquad 9\cancel{x}-21y=48$$

$$-9x+5y=16 \qquad \Longrightarrow \qquad \underline{-\cancel{9}x+5y=16}$$

$$-16y=64$$

$$y=-4$$

Sub -4 for y:

$$-3x+7(-4)=-16$$

$$-3x-28=-16$$

$$-3x-\cancel{28}+\cancel{28}=-16+28$$

$$-3x=12$$

$$x=-4$$

Solution: $(-4,-4)$.