3.1 Basic Differentiation Rules

Standards:

MCD1

MCD1e

$\boxed{\text{old}}$ Derivatives using Limit Definition

Find $f'(x)$ if $f(x) = x^2+5$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2+5]-[x^2+5]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+5}\cancel{-x^2}\cancel{-5}}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x+h = 2x+(0) = 2x$$

Computing this derivative isn't difficult. (Kinda easy but lots of work!) -_-

$\boxed{\text{new}}$ Basic Differentiation Rules

What if we considered such functions as $f(x) = x^4+3x^2+2$ or $g(x) = 5x^5+6x^3+x^4+3x^2$?

Computing the derivative of these functions is <u>EXTREMELY</u> DIFFICULT & TEDIOUS using the "limit method". Fortunately, several formulas have been developed to simplify the process of differentiation.

A Constants

Let's consider the function: $f(x) = c$, where c is some real number. Find $f'(x)$.

proof:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{c - c}{h}$$

$$= \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0$$

$$= 0.$$

$\blacksquare$

Conclusion

So, derivatives of all constants are 0.

$$\boxed{\begin{aligned} f(x) &= c \\ f'(x) &= 0. \end{aligned}}$$

B Linear Functions

Let's consider the function: $f(x) = nx$, where $n \in \mathbb{R}$. Find $f'(x)$.

proof:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[n(x+h)] - [nx]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{nx} + nh - \cancel{nx}}{h}$$

$$= \lim_{h\to 0} \frac{n\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} n$$

$$= n$$

Conclusion

The derivative of linear functions is the coefficent of the variable.

$$\boxed{\begin{aligned} f(x) &= nx \\ f'(x) &= n \end{aligned}}$$

BINOMIAL THEOREM

$$(x+h)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$= x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \binom{n}{3} x^{n-3} h^3 + \cdots + \binom{n}{n-1} x h^{n-1} + \binom{n}{n} h^n$$

$$= x^n + n x^{n-1} h + \frac{n(n-1)}{2!} x^{n-2} h^2 + \frac{n(n-1)(n-2)}{3!} x^{n-3} h^3 + \cdots$$

$$+ n x h^{n-1} + h^n$$

Where, $\binom{n}{k} = \frac{n}{k!(n-k)!}$.

$\boxed{C}$ Power Function

Let's consider the function: $f(x) = x^n$, where $n \in \mathbb{R}$.

proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^n] - [x^n]}{h}$$

$$= \lim_{h \to 0} \frac{\left[x^n + nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^3 + \dots + h^n\right] - [x^n]}{h}$$

$$= \lim_{h \to 0} nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2 + \dots + h^{n-1}$$

$$= \lim_{h \to 0} nx^{n-1}$$

$$= nx^{n-1}$$

∎

Conclusion

$$\boxed{\begin{aligned} f(x) &= x^n \\ f'(x) &= nx^{n-1} \end{aligned}}$$

[Examples] Find derivatives

① $f(x)=x^5$

$f'(x)=5x^4$

② $f(x)=x^{121}$

$f'(x)=121x^{120}$

③ $f(x)=x^{-2}$

$f'(x)=-2x^{-3}$

④ $f(x)=\frac{1}{x^5}\underset{\text{"Rewrite"}}{=}x^{-5}$

$f'(x)=-5x^{-6}$

⑤ $f(x)=\sqrt[3]{x}\underset{\text{"Rewrite"}}{=}x^{1/3}$

$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$