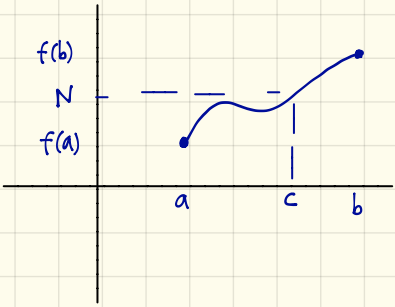
3.1 Intermediate Value Theorem

Intermediate Value Theorem

Suppose $f(x)$ is continuous on $[a,b]$ and suppose N is a number between $f(a)$ and $f(b)$: $f(a) < N < f(b)$.



Then, there is a number c in $[a,b]$ such that $f(c) = N$.

[Example] If $f(x) = \sqrt[3]{x} - 1 + x$. Prove that $f(x)$ has a root in the interval in the interval $(0,1)$.

- $f(0) = -1$ and $f(1) = 1$.
- root is when y-value is 0.

So, $f(0) < 0 < f(1)$.
Since $f(0) < 0 < f(1)$, IVT says that there is a number c in interval $(0,1)$ such that $f(c) = 0$.

[Example 2] Prove there is at least 1 solution to $\cos x = x^2$ in $(0,1)$.

$\cos x = x^2$
$\cos x - x^2 = f(x)$.

- $f(0) = \cos(0) - (0)^2 = 1$
- $f(1) = \cos(1) - (1) < 0$

So, $f(0) < 0 < f(1)$.
Since $f(0) < 0 < f(1)$, IVT says that there is a number c in interval $(0,1)$ such that $f(c) = 0$.