### 3.1 Intermediate Value Theorem

Intermediate Value Theorem
Suppose $f(x)$ is continuous on $[a, b]$ and suppose $N$ is a number between $f(a)$ and $f(b): \quad f(a)<N<f(b)$.


Then, there is a number $c$ in $[a, b]$ such that $f(c)=N$.
[Example] If $f(x)=\sqrt[3]{x}-1+x$. Prove that $f(x)$ has a root in the interval in the inter al $(0,1)$.

- $f(0)=-1$ and $f(1)=1$.
- root is when $y$-value is 0 .

So, $f(0)<0<f(1)$.
Since $f(0)<0<f(1)$, IVT says that there is a number $c$ in interval $(0,1)$ such that $f(c)=0$.
[Example 2] Prove there is at least 1 solution to $\cos x=x^{2}$ in $(0,1)$.

$$
\begin{array}{ll}
\cos x=x^{2} & \cdot f(0)=\cos (0)-(0)^{2}=1 \\
\cos x-x^{2}=f(x) . & \cdot f(1)=\cos (1)-(1)<0
\end{array}
$$

So, $f(0)<0<f(1)$.
Since $f(0)<0<f(1)$, IVT says that there is a number $c$ in interval $(0,1)$ such that $f(C)=0$.
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