3.1 Intermediate Value Theorem



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Intermediate Value Theorem

Suppose f(x) is continuous on [a, b] and suppose N is a number between f(a) and f(b): f(a) < N < f(b).



[Example] If $f(x) = \sqrt{x^2 - 1 + x}$. Prove that f(x) has a root in the interval in the interval (0, 1).

• f(0)=-1 and f(1)=1. • root is when y-value is 0.

So, f(0) < 0 < f(1). Since f(0) < 0 < f(1), IVT says that there is a number c in interval (0,1) such that f(c) = 0.

[Example 2] Prove there is at least 1 solution to $\cos x = x^2$ in (0,1)

 $\begin{array}{c} \cos x = x^{2} & \circ f(o) = \cos(o) - (o)^{2} = 1 \\ \cos x - x^{2} = f(x) & \circ f(1) = \cos(1) - (1) < 0 \end{array}$

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