3.2 Product Rule

Standard:

MCD1e

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Old Basic Differentiation Rules

① $f(x) = x^4 + 5$
$f'(x) = 4x^3$

② $f(x) = \frac{1}{x^2} = x^{-2}$
$f'(x) = -2x^{-3}$

③ $f(x) = \sqrt{10} + \sqrt{x} + \frac{5}{x^3}$
$= \sqrt{10} + x^{1/2} + 5x^{-3}$
$f'(x) = \frac{1}{2}x^{-1/2} - 15x^{-4}$

New Product Rule

Recall: What is the derivative of $f(x) = 2x^2 + 5x$

$f'(x) = 4x + 5$

• $\frac{d}{dx}(x^4 + 3x^2 + e^2) = 4x^3 + 6x$

Let's consider the function: $f(x) = (x+2)(5x)$.

What happens when we have a product of 2 functions?

One might be tempted to just take the derivative of each & then multiply just like one would for sum & difference of 2 functions.

However, this is WRONG!

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Let's prove what we need to do in this case.

PRODUCT RULE Let's consider the function: $k(x) - f(x) \cdot g(x)$.

Find the derivative.

proof:

$$k'(x) = \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\,g(x+h) - f(x+h)\,g(x) + f(x+h)\,g(x) - f(x)\,g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\left[g(x+h) - g(x)\right] + g(x)\left[f(x+h) - f(x)\right]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\left[g(x+h) - g(x)\right]}{h} + \lim_{h \to 0} \frac{g(x)\left[f(x+h) - f(x)\right]}{h}$$

$$= \lim_{h \to 0} f(x+h) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\boxed{= f(x) \cdot g'(x) + g(x) \quad f'(x)}$$

∎

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$\boxed{\text{Product Rule}}$ $f(x)g(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

[Example 1] $\frac{d}{dx}(x+2)(5x)$?

$f \cdot g' + g \cdot f'$ $= (x+2)(5) + (5x)(1)$

$= 5x + 10 + 5x$

$= 10x + 10$

[Example 2] $f(x) = (x^2+1)(x^3+1)$

$f'(x) = (x^2+1)(3x^2) + (x^3+1)(2x)$

$= 3x^4 + 3x^3 + 2x^4 + 2x$

$= 5x^4 + 3x^3 + 2x$

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