5.1 Introduction to Polynominals, Polynomial Operations

Standard:
A.APR.1

Old-A Types of Functions

① $y = 5x - 10$

- Linear Function
- Graphically is a line

② $y = 2^x - 10$

- Exponential Function
- Graphically is a curve

new-A Polynomial Operations

- Poly = many -nomial = terms

What is a Polynomial?

- An expression (or string of terms) that can have constants & variables with whole number (positive) exponents.

Example) $5x^2 - 3y + 10$
$-2y^3 - 9$

Nonexamples) $x + y^{-2}$
$5x^{\frac{1}{2}} - \frac{2}{x}$

- Polynomials are named according to their degree & number of terms.

Degree The highest exponent of a polynomial.

Degree	Name	Example
0	Constant	3
1	Linear	$2x + 8$
2	Quadratic	$3x^2 + 2x - 5$
3	Cubic	$10x^3$
4	Quartic	$6x^4 - 8x^2$
5	Quintic	$-2x^5 + x^3 + x$
6	6th Degree etc.	$4x^6 - 7x^4 + 8$

Number of Terms A string of expressions separated by plus, minus signs.

Terms	Name	Example
1	Monomial	$3x$
2	Binomial	$2y + 8$
3	Trinomial	$8x^2 + 5x - 2$
4	Polynomial	$6x^5 - 7x^4 + 4x - 1$

[Example 1] Name the Polynomials.

① $-7+3n^3$ Cubic Binomial
② $5n$ Linear Monomial
③ $-x^4+3x^2-11$ Quartic Trinomial

[old-B] Combining Like Terms

Simplify the expressions.
① $3x-6+2x-8 = 5x-14$
② $3x-7+12x+10 = 15x+3$

[new-B] Polynomial Operations (Add & Subtract)

Let's consider Michael's report card for school. Let's say he made the following subject grades:

AP Environment - B
Math - A
Fitness - A
American Gov't - C
English - B
Spanish - A

Report out on Michael's grades.

Michael has 3 A's, 2 B's and 1 C.

conclusion: We added grades that were "alike".

Adding Polynomials ⟹ Drop the Parenthesis & Combine Like Terms.

[Example 1]

$(2x^2-4x+3)+(x^2+5x-1)$

$=2x^2-4x+3+x^2+5x-1$

$=2x^2+x^2-4x+5x+3-1$

$=3x^2+x+2$

[Example 2]

$(6+x^3+3x)+(2x-8)$

$=6+x^3+3x+2x-8$

$=x^3+3x+2x-8+6$

$=x^3+5x-2$

Subtracting Polynomials ⟹ Distribute the negative & Combine Like Terms

[Example 3]

$(3a^4+10a^2)-(8a^2-a^4+6)$

$=3a^4+10a^2-8a^2+a^4-6$

$=3a^4+a^4+10a^2-8a^2-6$

$=4a^4+2a^2-6$

[Example 4]

$(3m^7+2m^3-4m)-(2m^3-m+7)$

$=3m^7+2m^3-4m-2m^3+m-7$

$=3m^7+2m^3-2m^3-4m+m-7$

$=3m^7-3m-7$

[Example 5] Find the perimeter of the rectangle.

Perimeter is the sum of all sides.

$\text{Perimeter}=(3x+7)+(2x-1)+(3x+7)+(2x-1)$

$=3x+7+2x-1+3x+7+2x-1$

$=3x+2x+3x+2x+7-1+7-1$

$=10x+12$

Old - C Distributive Property

① $5(7x-6) = 35x-30$

② $4(2-4x)-6(x-4) = 8-16x-6x+24$

$= 8+24-16x-6x$

$= 32-22x$

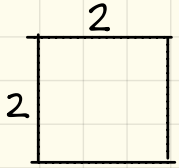
$= -22x+32$

Remember Imaginary 1 Rule:

x means $1x$, x^2 and $1x^1$.

New - C Polynomial Operations (Multiply)

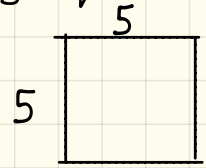
Let's consider the following squares. Find the area.



Area $= 2 \cdot 2$

$= 2^2$

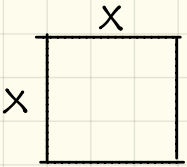
$= 4$



Area $= 5 \cdot 5$

$= 5^2$

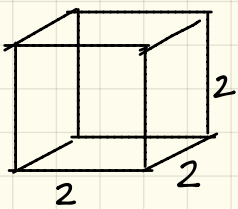
$= 25$



Area $= x \cdot x$

$= x^2$

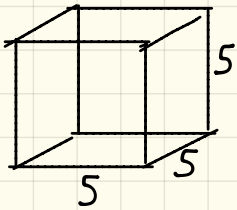
Let's consider the following cubes. Find the surface area.



Surface Area $= 2 \cdot 2 \cdot 2$

$= 2^3$

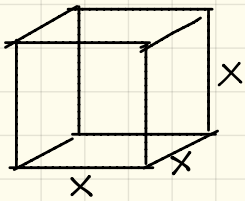
$= 8$



Surface Area $= 5 \cdot 5 \cdot 5$

$= 5^3$

$= 125$



Surface Area $= x \cdot x \cdot x$

$= x^3$

Conclusion

$x \cdot x = x^2$

$x \cdot x \cdot x = x^3$

When multiplying variables, you add the exponents.

Exponential Rule: $x^a \cdot x^b = x^{a+b}$

Multiplying Polynomials $\Longrightarrow$ Distribute, follow exponent rules & combine like terms

Case A Monomial times Binomial

[Example 6]

$x^3(7x+6) = 7x^4+6x^3$

[Example 7]

$4x(x^4+2x-10) = 4x^5+8x^2-40x$

[Example 8]

$-2x(x^2-4x+2) = -2x^3+8x^2-4x$

Case B Binomial times Binomial $\rightsquigarrow$ use FOIL Method

First
Outer
Inner
Last

[Example 9]

$(x+3)(x-3) = x^2-3x+3x-9$

$= x^2-9$

[Example 11]

$(x+2)(x-3) = x^2-3x+2x-6$

$= x^2-1x-6$

[Example 10]

$(3x-1)(2x-4) = 6x^2-12x-2x+4$

$= 6x^2-14x+4$

[Example 12]

$(2x-1)(x^2+7) = 2x^3+14x-1x^2-7$

$= 2x^3-x^2+14x-7$