5.1 Optimization

Standards:

MCA3

MCA3c

$\boxed{0d}$ First Derivatives Test

The first derivatives test gives
$\boxed{1}$ local max & local min values
$\boxed{2}$ intervals of increase/decrease
$\boxed{3}$ critical values

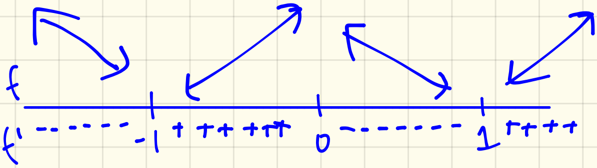
[Example 1] $f(x) = x^4 - 2x^2$

$f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0, -1, 1$



Intervals of Increase: $(-1, 0) \cup (1, \infty)$

Intervals of Decrease: $(-\infty, -1) \cup (0, 1)$

Local Maxs: $(0, f(0)) = (0, 0)$

Local Mins: $(-1, f(-1)) = (-1, -1)$

$(1, f(1)) = (1, -1)$

[Example 2] $f(x) = x^3 - x$

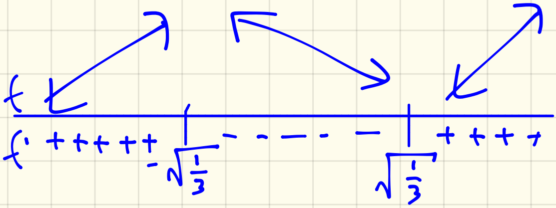
$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$



Intervals of Increase: $(-\infty, -\sqrt{\frac{1}{3}}) \cup (\sqrt{\frac{1}{3}}, \infty)$

Intervals of Decrease: $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$

Local Max: $(-\sqrt{\frac{1}{3}}, f(-\sqrt{\frac{1}{3}}))$

Local Min: $(\sqrt{\frac{1}{3}}, f(\sqrt{\frac{1}{3}}))$

$\boxed{\text{new}}$ Optimization

These are word problems that involve finding a maximum or minimum value.

Tips for Modeling Optimization Problems

1. Determine the quantity or function to be optimized.
2. If possible, make sketches showing how the elements that vary are related. Label your sketches clearly by assigning variables to quantities which change.
3. Obtain a formula for the function to be optimized in terms of the variable you identified in previous step. If possible, eliminate all but one variable from the formula.
4. Find all critical points and evaluate the function at these points and the endpoints (if relevant) to find global maxima & minima.

[Example 1] Find 2 numbers whose difference is 100 and whose product is a minimum.

Let x & y be the numbers.

$x - y = 100$ (constraint equation)

xy (the objective)

$\boxed{\text{Note:}}$ Use the constraint eqn. to write the objective in terms of one variable only.

Rewrite constraint:

$x - y = 100$

$y = x - 100.$

Manipulate Objective:

xy

$x(x-100).$

$f(x) = x(x-100)$

$\quad = x^2 - 100x$

Check answer by 2nd Deriv. Test!

$f''(x) = 2 > 0$ gives minimum.

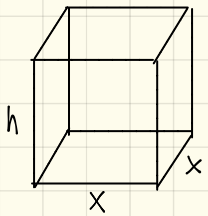
$f'(x) = 2x - 100 = 0$

$\boxed{x = 50}.$

$y = 50 - 100$

$\boxed{y = -50}.$

[Example 2] A box with a square base & an open top must have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of material used.



Let x be the base & width.
Let h be the height.

Constraint: Volume = 32000 $\quad (x^2h = 32000)$
Objective: minimize surface area

$$SA = x^2 + 4xh$$

Rewrite constraint:

$$x^2h = 32000$$
$$h = \frac{32000}{x^2}$$

Manipulate Objective:

$$f(x) = x^2 + 4xh$$
$$= x^2 + 4x\left(\frac{32000}{x^2}\right)$$
$$= x^2 + \frac{128000}{x}$$

Find x:

$$f(x) = x^2 + \frac{128000}{x}$$
$$= x^2 + 128000x^{-1}$$

$$f'(x) = 2x - 128000x^{-2}$$
$$= 2x - \frac{128000}{x^2} = 0$$

$$\frac{2x^3}{x^2} - \frac{128000}{x^2} = 0$$

$$2x^3 - 128000 = 0$$
$$2x^3 = 128000$$
$$x^3 = 64000$$
$$\boxed{x = 40}$$

need to find h:

$$h = \frac{32000}{40^2}$$
$$\boxed{h = 20}$$

This was created by Keenan Xavier Lee, 2013. See my website for more information, lee-apcalculus.weebly.com.