5.3 Slope Fields

Standards:

MCA3

MCA3a

MC11

MC11a

$\boxed{\text{old}}$ Related Rates

A spherical snowball melts in such a way that the instant at which its radius is 20 cm, its radius is decreasing at 3 cm/min. At what rate is the volume of the ball changing at that instant?

given: $r = 20\,cm$, $\frac{dr}{dt} = -3 \frac{cm}{min}$, $\frac{dV}{dt}$

Eqtn: $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi (20\,cm)^2 \cdot \left(-3 \frac{cm}{min}\right)$$

$$= -4800\pi \frac{cm^3}{min}$$

$\boxed{\text{new}}$ Slope Fields

Let's consider the following situation:
Suppose we are interested in how fast an employee learns at a given task.

rate a person learns = percentage of task not yet learned.

Let y = the percentage learned by time (t weeks).

Equation: $\frac{dy}{dt} = 100 - y$ $\Longrightarrow$ an example of differential equation.

Differential Equation – is such an equation that gives information about the rate of change of an unknown function.

Let's say that work is a 5 day work weekand let's assume 100% of the learning rate holds for the first day. How much does the person need to learn in 1 day?

$$\frac{dy}{dt} = 100 - y$$

(1 day = 1/5 of wk)

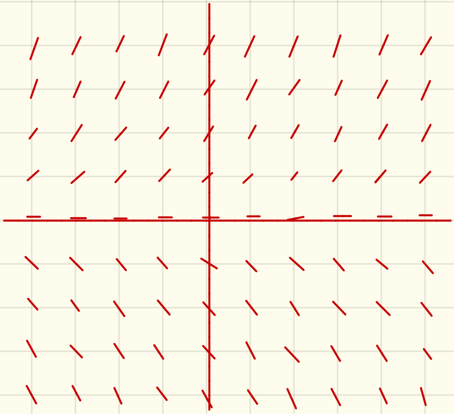
$$= 100 - (20\%) = 80\% \text{ per day}$$

We can also visualize differential equation using SLOPE FIELDS!

Let's consider the differential equation:

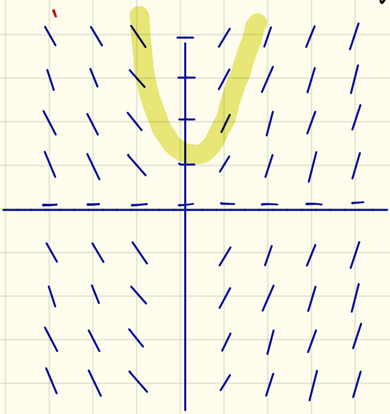
$$\frac{dy}{dx} = y$$

Any solution to the differential eqtn. has the property that the slope of any point is equal to the y-coordinate.



x	y	$\frac{dy}{dx} = y$ → slopes
0	0	0
1	1	1
1	3	3
2	-3	-3
0	1	1
0	-1	-1

Let's consider the differential equation: $\frac{dy}{dx} = 2x$. Sketch the slope field.



x	y	$\frac{dy}{dx}=2x$
0	0	0
1	1	2
2	4	4
-2	4	-4
-1	-1	-2

Let's find the particular solution to the equation $\frac{dy}{dx} = 2x$ whose graph passes through $(1,2)$

$$\frac{dy}{dx} = 2x$$
$$dy = 2x\,dx$$
$$\int dy = \int 2x\,dx$$
$$y = x^2 + C \leftarrow \text{antiderivative}$$

When $x=1$, $y=2$

$$y = x^2 + c$$
$$2 = (1)^2 + c$$
$$2 = 1 + c$$
$$1 = c$$

$$y = x^2 + 1$$