5.4 Solving Quadratics, Part 1

GCF Factoring, Difference of Squares & Factoring Trinomials

Standards:

A.SSE.3a

A.RE1.4

Old-A Factoring

Factor the expressions.

① $x^2-7x-18$ (1·18, 2·9)
$=(x-2)(x-9)$

② $7k^2+9k$
$=k(7k+9)$

GCF of #'s: 7, 9 — ①7, ①9
GCF of variable: K^2, K — (K)K, (K)

③ $p^2-5p-14$ (1·14, 7·2)
$=(p-7)(p+2)$

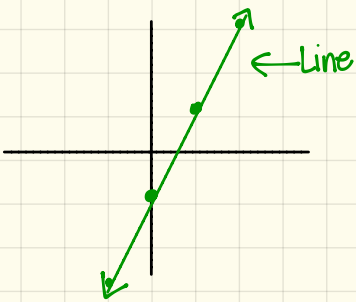
④ $7x^2-31x-20$ (1·7; 1·20, 2·10, 4·5)
$=(7x+5)(x-4)$
$-5x$
$-28x$

⑤ x^2-49
$=(x-7)(x+7)$

⑥ $2m^2-18m+16$ (1·8, 2·9)
$2(m^2-9m+8)$
$2(m-8)(m-1)$

Old-B Linear & Exponential Functions (Algebraic / Graphic)

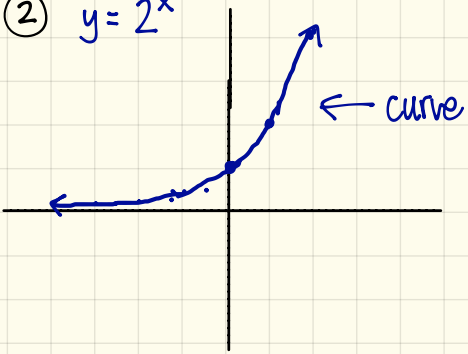
① $y=2x-1$ $m=2, b=-1$



x	y
-2	-5
-1	-3
0	-1
1	1
2	3
⋮	⋮

Parent Function of Linear Function
$y=mx+b$

② $y = 2^x$



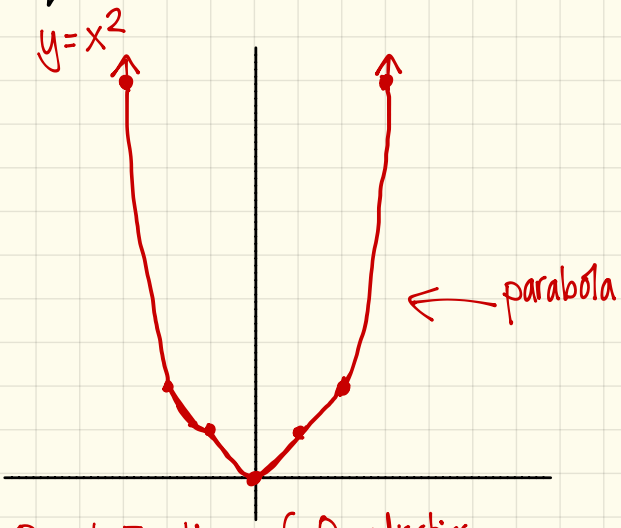
x	y
0	1
1	2
2	4
3	8
4	16
⋮	⋮

Parent Function of Exponentials

$y = a^x$

new-A Quadratics

Let's consider the equation $y = x^2$. Create a graph & table of the equation.



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
⋮	⋮

Parent Function of Quadratics

$y = x^2$

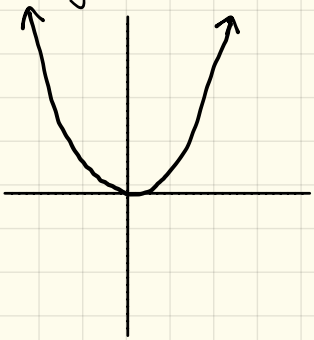
new-B Solving Quadratics

What does it mean to solve Quadratics?

Solving Quadratics means to find the x-intercepts of the Quadratic function.

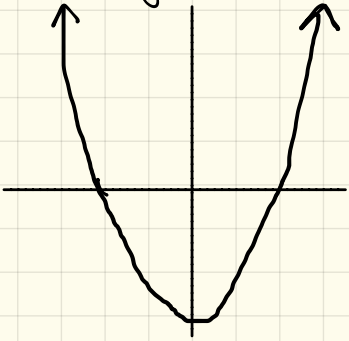
[Examples] Solve the Quadratics (Graphically)

① $y = x^2$



x-intercept: (0,0)

② $y = x^2 - 4$



x-intercepts: (-2,0), (2,0)

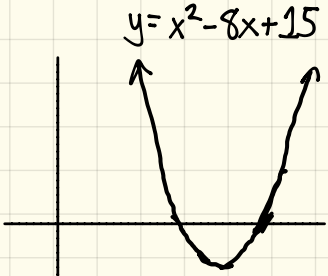
③ $y = x^2 - 8x + 15$

x-intercepts: (3,0), (5,0)

Solving Quadratics Algebraically

Let's consider the graph of the equation of $y = x^2 - 8x + 15$.

How do we find the x-intercepts of this graph algebraically?



Finding the intersecting points between the 2 lines $y = x^2 - 8x + 15$ & $y = 0$ (x-axis).

$y = x^2 - 8x + 15$ ← graph

$y = 0$ ← x-axis (System of equations)

$y = x^2 - 8x + 15$ Use substitution method to solve.
$y = 0$

$x^2 - 8x + 15 = 0$ (1·15, 3·5) Solve for x.
$(x-3)(x-5) = 0$ Use the zero product property to solve for x.

$x - 3 = 0$ $\quad$ $x - 5 = 0$
$+3 = +3$ $\quad$ $+5 = 5$
$x = 3$ $\quad$ $x = 5$

The x-intercepts are (3,0) and (5,0).

Zero product Property
If $a \cdot b = 0$, then $a = 0$ or $b = 0$ (or both $a = 0$ and $b = 0$).

Conclusion One way to solve quadratics is to:
1. set quadratic equation equal to 0
2. factor quadratic
3. use zero product property.

[Examples] Solve for x.

Case A Solving Quadratics – Factoring Trinomials (set equal to 0)

① $x^2 + 3x - 18 = 0$ (1·18, 2·9, 3·6)
$(x+6)(x-3) = 0$
$x + 6 = 0$ or $x - 3 = 0$
$x = -6$ $\quad$ $x = 3$

② $x^2 + 10x + 16 = 0$ (1·16, 2·8, 4·4)
$(x+2)(x+8) = 0$
$x + 2 = 0$ or $x + 8 = 0$
$x = -2$ $\quad$ $x = -8$

Case B Solve Quadratics — Difference of Squares (set equal to 0)

③ $x^2 - 49 = 0$

$x^2 + 0x - 49 = 0$ (1·49, −7·7)

$(x+7)(x-7) = 0$

$x+7=0$ or $x-7=0$

$x = -7$ or $x = 7$

④ $x^2 - 36 = 0$

$(x+6)(x-6) = 0$

$x+6=0$ or $x-6=0$

$x = -6$ or $x = 6$.

Case C Solve Quadratics — GCF Factoring (set equal to 0)

⑤ $x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$ or $x+4=0$

$x = 0$ or $x = -4$.

⑥ $5x^2 + 10x = 0$

$5x(x+2) = 0$

$5x = 0$ or $x+2=0$

$x = 0$ or $x = -2$.

⑦ $4x^2 - 16x = 0$

$4x(x-4) = 0$

$4x = 0$ or $x-4=0$

$x = 0$ or $x = 4$.

⑧ $2x^2 - 14x + 12 = 0$

$2(x^2 - 7x + 6) = 0$

$2(x-1)(x-6) = 0$

$x-1=0$ or $x-6=0$

$x = 1$ or $x = 6$.

[More Examples] Solve for x.

⑨ $2x^2 - 5x + 2 = 0$ (1·2, 1·2)

$(2x-1)(x-2) = 0$

$2x-1=0$ or $x-2=0$

$+1=+1$

$2x = 1$

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

$x = 2$

⑩ $x^2 - 6x = 9$ Set equal to 0.

$-9 = -9$

$x^2 - 6x - 9 = 0$ Solve for x.

$(x-3)(x-3) = 0$

$x-3=0$ or $x-3=0$

$x = 3$

(11) $x^2 - 3 = -2x$

$+2x \quad +2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x+3=0$ or $x-1=0$

$x=-3 \qquad x=1.$

(12) $3x^2 = -9x$

$+9x = +9x$

$3x^2 + 9x = 0$

$3x(x+9) = 0$

$3x=0$ or $x+9=0$

$x=0$ or $x=-9$

(13) $3x^2 = 27$

$-27 = -27$

$3x^2 - 27 = 0$

$3(x^2 - 9) = 0$

$3(x+3)(x-3) = 0$

$x+3=0$ or $x-3=0$

$x=-3$ or $x=3.$

WAYS to SOLVE QUADRATICS

(Quad & Linear or Constant) 2 Terms

- GCF Factoring (set = 0)
- Difference of Squares (set = 0)

3 Terms (Quad, Linear, Constant)

- GCF Factoring (set = 0)
- Factoring Trinomials (set = 0)