5.6 Solving Quadratics, Part 3

Completing the Square & Quadratic Formula

Standards:

A.REI.4

A.REI.4a

A.REI.4b

$\boxed{\text{old}}$ Solving Quadratics – Taking the Square Root

① $(x-4)^2 = 5$

$\sqrt{(x-4)^2} = \sqrt{5}$

$x - 4 = \pm\sqrt{5}$

$+4 = +4$

$x = 4 \pm \sqrt{5}$

$x = 4+\sqrt{5}$ or $x = 4-\sqrt{5}$

② $(x-2)^2 + 2 = 18$

$-2 = -2$

$(x-2)^2 = 16$

$\sqrt{(x-2)^2} = \sqrt{16}$

$x - 2 = \pm 4$

$+2 = +2$

$x = 2 \pm 4$

$x = 2+4$ or $x = 2-4$

$x = 6$ or $x = -2$

WAYS TO SOLVE QUADRATICS

(Quad & Linear or Constant) 2 Terms

- GCF Factoring (set = 0)
- Difference of Squares (set = 0)
- Taking Sq Root (isolate Quad term)

3 Terms (Quad, Linear, Constant)

- GCF Factoring (set = 0)
- Factoring Trinomials (set = 0)

$\boxed{\text{new-A}}$ Solving Quadratics – Completing the Square

Let's consider the equation $x^2 + 6x + 7 = 0$. Factor the equation.

$x^2 + 6x + 7 = 0$ — $1 \cdot 7$

$\Longrightarrow$ This Quadratic Equation is not <u>factorable.</u>

How do we solve a Quadratic <u>Trinomial</u> that is not factorable?

To use the "Completing the Square" method, you must:

- Move the constant term to the other side of equal sign
- Add $\left(\frac{b}{2}\right)^2$ to both sides of the equal sign
- Factor!
- Take the Square Root on both sides
- Solve for x.

Let's consider $x^2+6x+7=0$. Solve for x.

$$x^2+6x+7=0$$
$$x^2+6x=-7$$
$$x^2+6x+\left(\frac{6}{2}\right)^2=-7+\left(\frac{6}{2}\right)^2$$
$$x^2+6x+9=-7+9$$
$$x^2+6x+9=2$$
$$(x+3)(x+3)=2$$
$$(x+3)^2=2$$
$$\sqrt{(x+3)^2}=\sqrt{2}$$
$$x+3=\pm\sqrt{2}$$
$$-3=-3$$
$$x=-3\pm\sqrt{2}$$
$$x=-3+\sqrt{2} \text{ or } x=-3-\sqrt{2}$$

[Examples]

① $a^2+2a-3=0$

$$a^2+2a=3$$
$$a^2+2a+\left(\tfrac{2}{2}\right)^2=3+\left(\tfrac{2}{2}\right)^2$$
$$a^2+2a+1=3+1$$
$$(a+1)(a+1)=4$$
$$(a+1)^2=4$$
$$\sqrt{(a+1)^2}=\sqrt{4}$$
$$a+1=\pm 2$$
$$-1 \quad -1$$
$$a=-1\pm 2$$
$$a=-1+2 \text{ or } a=-1-2$$
$$a=1 \text{ or } a=-3$$

② $m^2-12m+26=0$

$$m^2-12m=-26$$
$$m^2-12m+\left(\tfrac{12}{2}\right)^2=-26+\left(\tfrac{12}{2}\right)^2$$
$$m^2-12m+36=-26+36$$
$$(m-6)(m-6)=10$$
$$(m-6)^2=10$$
$$\sqrt{(m-6)^2}=\sqrt{10}$$
$$m-6=\pm\sqrt{10}$$
$$+6=+6$$
$$m=6\pm\sqrt{10}$$
$$m=6+\sqrt{10} \text{ or } m=6-\sqrt{10}$$

③ $x^2-18x-85=0$

$$x^2-18x=85$$
$$x^2-18x+\left(\tfrac{18}{2}\right)^2=85+\left(\tfrac{18}{2}\right)^2$$
$$x^2-18x+81=85+81$$
$$(x-9)(x-9)=166$$
$$(x-9)^2=166$$
$$\sqrt{(x-9)^2}=\sqrt{166}$$
$$x-9=\pm\sqrt{166}$$
$$+9=+9$$
$$x=9\pm\sqrt{166}$$
$$x=9+\sqrt{166} \text{ or } 9-\sqrt{166}$$

④ $x^2+6x-90=8$

$$x^2+6x=98$$
$$x^2+6x+\left(\tfrac{6}{2}\right)^2=98+\left(\tfrac{6}{2}\right)^2$$
$$x^2+6x+9=107$$
$$(x+3)(x+3)=107$$
$$(x+3)^2=107$$
$$\sqrt{(x+3)^2}=\sqrt{107}$$
$$x+3=\pm\sqrt{107}$$
$$-3=-3$$
$$x=-3\pm\sqrt{107}$$
$$x=-3+\sqrt{107} \text{ or } x=-3-\sqrt{107}$$

WAYS to SOLVE QUADRATICS

(Quad & Linear or Constant) 2 Terms	3 Terms (Quad, Linear, Constant)
· GCF Factoring (set = 0)	· GCF Factoring (set = 0)
· Difference of Squares (set = 0)	· Factoring Trinomials (set = 0)
· Taking Sq Root (isolate Quad term)	· Completing the Square (move c)

$\boxed{\text{new - B}}$ Quadratic Formula

Let's consider the standard form of Quadratics Equation: $ax^2+bx+c=0$

Let's use the "Complete the Square" method to solve for x.

$$ax^2+bx+c=0$$
$$ax^2+bx=-c$$
$$\frac{\cancel{a}x^2}{\cancel{a}}+\frac{bx}{a}=-\frac{c}{a}$$
$$x^2+\frac{bx}{a}=\frac{-c}{a}$$
$$x^2+\frac{bx}{a}+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$
$$x^2+\frac{bx}{a}+\frac{b^2}{4a^2}=\frac{-c}{a}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$
$$\sqrt{\left(x+\frac{b}{2a}\right)^{\cancel{2}}}=\sqrt{\frac{b^2-4ac}{4a^2}}$$
$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

QUADRATIC FORMULA

$$ax^2+bx+c=0$$ $\leftarrow$ must be set equal to 0.

$\boxed{\text{Quadratic Formula}}$ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- Label a, b & c from Quadratic Equation
- Substitute a, b & c values into formula
- Compute x.

[Examples] Solve for x.

① $x^2-6x+3=0$

$a=1$
$b=-6$
$c=3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(3)}}{2(1)}=\frac{6\pm\sqrt{24}}{2}$$

$$=\frac{6\pm2\sqrt{6}}{2} \qquad x=\frac{6}{2}+\frac{2\sqrt{6}}{2} \text{ or } x=\frac{6}{2}-\frac{2\sqrt{6}}{2}$$

$$=3+\sqrt{6} \text{ or } 3-\sqrt{6}$$

② $2x^2+2x-3=0$

$a=2$
$b=2$
$c=-3$

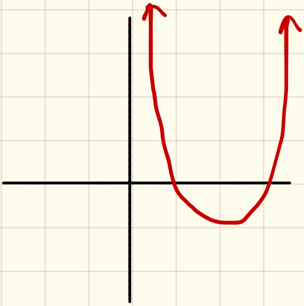
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{(2)^2-4(2)(-3)}}{2(2)}=\frac{-2\pm\sqrt{28}}{4}$$

$$=\frac{-2\pm2\sqrt{7}}{4}=\frac{-2}{4}\pm\frac{2\sqrt{7}}{4}=-\frac{1}{2}\pm\frac{\sqrt{7}}{2}.$$

$$=\frac{-1}{2}+\frac{\sqrt{7}}{2} \text{ or } \frac{-1}{2}-\frac{\sqrt{7}}{2}.$$

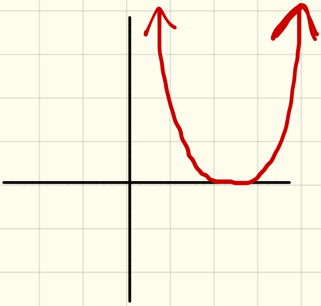
How many different solutions can a Quadratic Equation have graphically?

2 solutions



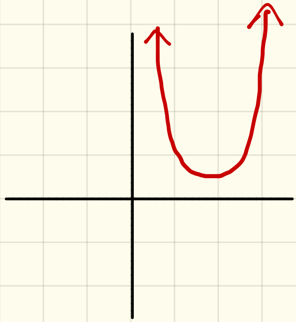
- Touches x-axis twice
- Two x-intercepts

1 solution



- Touches x-axis once
- One x-intercept

No solution



- Doesn't touch x-axis
- No x-intercept

How do we determine algebraically how many solutions a quadratic equation has?

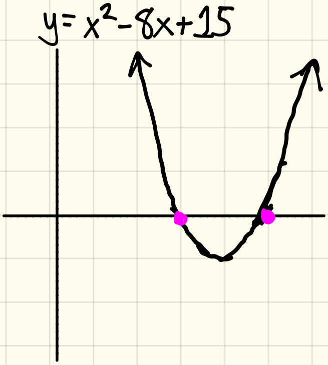
To determine the number of solutions for x algebraically, we use the discriminant.

$$\text{Discriminant} = b^2 - 4ac.$$

If the Discriminant is:

- positive — 2 real solutions
- zero — 1 real solution
- negative — 0 real solutions

Let's consider the graph of $y = x^2 - 8x + 15$.



How many solutions does $y = x^2 - 8x + 15$ have?

2 real solutions
(crosses the x-axis twice).

Find how solutions $y = x^2 - 8x + 15$ may have algebraically?

$a = 1$
$b = -8$
$c = 15$

$$
\begin{aligned}
\text{Discriminant} &= b^2 - 4ac \\
&= (-8)^2 - 4(1)(15) \\
&= 4
\end{aligned}
$$

Because the discriminant is positive, there are 2 real solutions.

[Examples] Determine the amount of solutions each Quadratic has.

1. $x^2 - 4x + 4 = 0$

$a = 1$
$b = -4$
$c = 4$

$$
\begin{aligned}
\text{Discriminant} &= b^2 - 4ac \\
&= (-4)^2 - 4(1)(4) \\
&= 0.
\end{aligned}
$$

Because the discriminant is zero, there is only 1 real solutions.

② $y = x^2 - 3x + 4 = 0$

$a = 1$
$b = -3$
$c = 4$

$$\text{Discriminant} = b^2 - 4ac$$
$$= (-3)^2 - 4(1)(4)$$
$$= -7$$

Because the discriminant is negative, there are no real solutions.