5.8 Characteristics of Quadratic Functions

with Rates of Change

Standards:

F.IF.1 F.IF.7a

F.IF.2 F.LE.3

F.IF.4

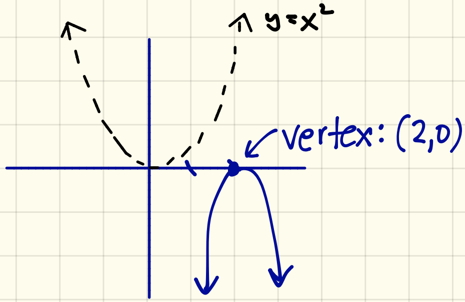
F.IF.6

F.IF.7

$\boxed{\text{old}}$ Transformations of Quadratics

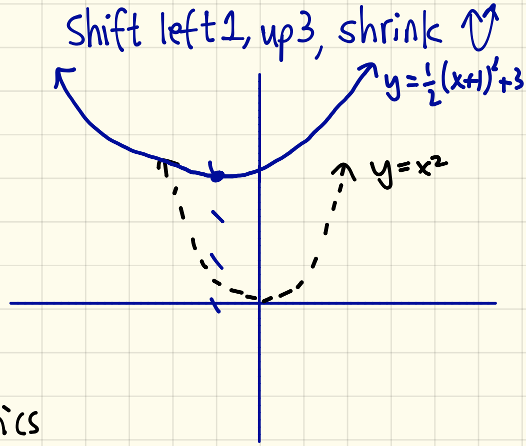
① $f(x) = -3(x-2)^2$

shift right 2, stretch,



② $f(x) = \frac{1}{2}(x+1)^2 + 3$

Shift left 1, up 3, shrink



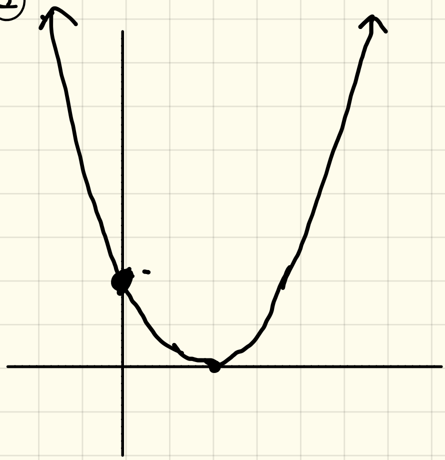
$\boxed{\text{new-A}}$ Characteristics of Quadratics

- Domain: the set of x-values (how far left to right the graph spans)
- Range: the set of y-values (how far down to up the graph spans)
- zeros: the x-intercept(s) of the graph (also called roots or solutions)
- y-intercept: the point where the graph crosses the y-axis
- Extrema: the maximum or minimum point, maximum minimum
- Interval of Increase: the set of x-values where the slopes are positive.
- Interval of Decrease: the set of x-values where the slopes are negative
- Axis of Symmetry: the line where the parabola is symmetric
- Vertex: the maximum or minimum point

[Examples] Answer each using the graph.

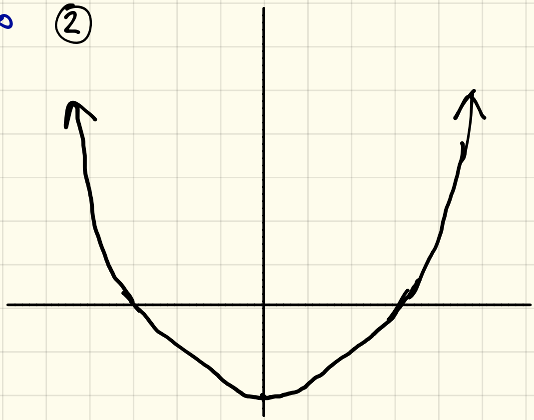
1a) Domain: $(-\infty,\infty)$ or $\mathbb{R}$ or $-\infty < x < \infty$
1b) Range: $[0,\infty)$ or $y \geq 0$
1c) Zeros: $(2,0)$
1d) y-intercept: $(0,2)$
1e) Extrema: $(2,0) \leftarrow$ minimum
1f) Interval of Increase: $(2,\infty)$
1g) Interval of Decrease: $(\infty,2)$
1h) Axis of Symmetry: $x=2$
1i) Vertex: $(2,0)$

1



2a) Domain: $(-\infty,\infty)$ or $\mathbb{R}$ or $-\infty < x < \infty$
2b) Range: $[-2,\infty)$ or $y \geq -2$
2c) Zeros: $(-3,0)$, $(3,0)$
2d) y-intercept: $(0,-2)$
2e) Extrema: $(0,-2)$
2f) Interval of Increase: $[0,\infty)$
2g) Interval of Decrease: $(\infty,0]$
2h) Axis of Symmetry: $x=0$
2i) Vertex: $(0,-2)$

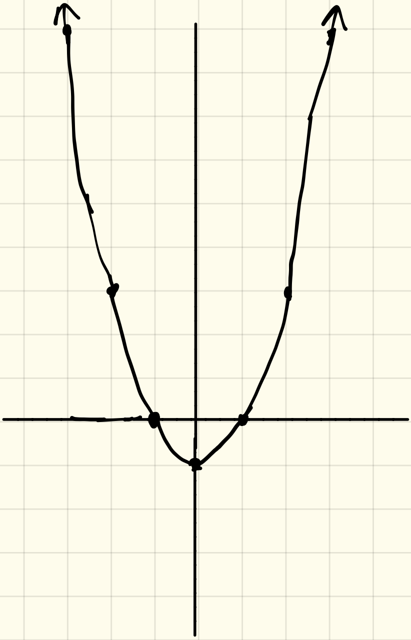
2



1a) Domain: $(-\infty,\infty)$ or $\mathbb{R}$ or $-\infty < x < \infty$
1b) Range: $(-\infty,2]$
1c) Zeros: $(-3,0)$, $(-1,0)$
1d) y-intercept: $(0,-4)$
1e) Extrema: $(-2,2)$
1f) Interval of Increase: $(-\infty,-2)$
1g) Interval of Decrease: $(-2,\infty)$
1h) Axis of Symmetry: $x=-2$
1i) Vertex: $(-2,2)$

new-B Quadratic Functions Rate of Change

Let's consider the function $f(x) = x^2 - 1$ Graph the function & determine the rate of change being asked.



ⓐ Find the rate change between $(0,-1)$ and $(2,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - {}^{-}1}{2 - 0} = \frac{4}{2} = 2$$

ⓑ Find the rate change between $(-3,8)$ and $(-2,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 8}{-2 - {}^{-}3} = \frac{-5}{1} = -5$$

- Quadratic Functions do not have constant rates of change.

Let's consider the three functions: $f(x) = 2x+1$, $g(x) = 2^x$, $h(x) = x^2+1$.
Who wins? Which function will have the highest y-value?

Let's use analytical approach to analyze.

x	f(x)	g(x)	h(x)
0	1	1	1
1	3	2	2
2	5	4	5
3	7	8	10
4	9	16	17
5	11	32	26
6	13	64	37
7	15	128	50
8	17	256	65
9	19	512	82
10	21	1024	101

Exponential begins to lead here!

Conclusion

1st place • Exponential Functions starts off slower, but eventually increasingly exceeds both Linear & Quadratic Functions.

2nd place • Quadratic Functions eventually incensing exceeds Linear Functions.

Last place • Linear Functions have a constant rate of change and will always eventually lose against Exponential & Quadratic Functions.

note: using a graphing calculator can to visually see the y-values!