5.9 Changing Vertex, Standard Forms

Moving from Vertex to Standard Form (and vice versa)

Standards:

A.SSE.3b

F.IF.8

F.IF.8a

F.IF.9

$\boxed{\text{Old}}$ Quadratics Transformations

Let's recall the Standard & Vertex Forms of Quadratics.

Standard Form

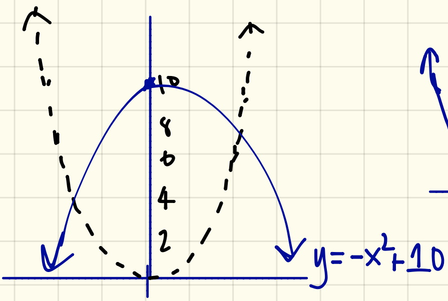
$y = ax^2 + bx + c$

Vertex Form

$y = a(x-h)^2 + k$

① $f(x) = -x^2 + 10$

shift down 10, ∩

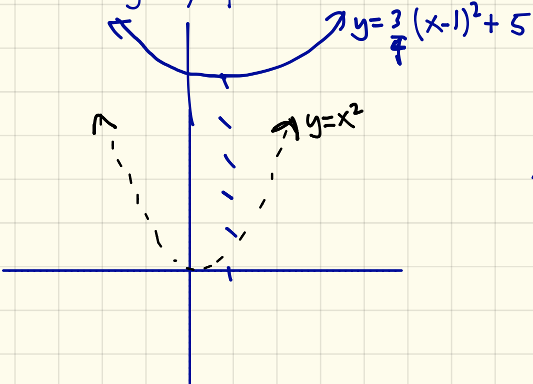


② $f(x) = (x+5)^2$

shift left 5, ∪

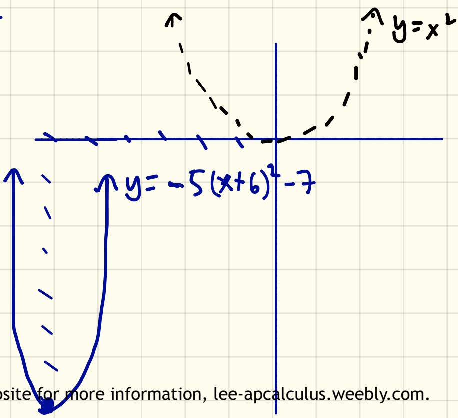
③ $f(x) = \frac{3}{4}(x-1)^2 + 5$

shift right 1, up 5, shrinks ∪



④ $f(x) = -5(x+6)^2 - 7$

shift left 6, down 7, stretch ∩



new Changing Forms

1 Standard to Vertex $\longrightarrow$ Use the Completing the Square Method

- Set the problem up to complete the square $\Rightarrow$ $y=ax^2+bx+c$
  (Factor out a number if needed.)
- Add & Subtract the square of half of the coefficient of the linear term.
- Combine like terms
- Write the perfect square trinomial as a binomial squared.
  $\Longrightarrow$ $y=a(x-h)^2+k$

[Examples] Convert Standard to Vertex

① $y=x^2+8x+10$

$$\begin{aligned}y&=(x^2+8x)+10\\&=(x^2+8x+\_)+10-\_\\&=\left(x^2+8x+\left(\tfrac{8}{2}\right)^2\right)+10-\left(\tfrac{8}{2}\right)^2\\&=(x^2+8x+16)+10-16\\&=(x+4)(x+4)-6\\&=(x+4)^2-6\end{aligned}$$

② $y=x^2+6x+8$

$$\begin{aligned}y&=(x^2+6x)+8\\&=(x^2+6x+\_)+8-\_\\&=\left(x^2+6x+\left(\tfrac{6}{2}\right)^2\right)+8-\left(\tfrac{6}{2}\right)^2\\&=(x^2+6x+9)+8-9\\&=(x+3)(x+3)-1\\&=(x+3)^2-1\end{aligned}$$

③ $f(x)=x^2-4x+3$

$$\begin{aligned}&=(x^2-4x)+3\\&=(x^2-4x+\_)+3-\_\\&=\left(x^2-4x+\left(\tfrac{4}{2}\right)^2\right)+3-\left(\tfrac{4}{2}\right)^2\\&=(x^2-4x+4)+3-4\\&=(x-2)(x-2)-1\\&=(x-2)^2-1\end{aligned}$$

④ $y=3x^2+24x+50$

$$\begin{aligned}&=(3x^2+24x)+50\\&=3(x^2+8x)+50\\&=3(x^2+8x+\_)+50-3(\_)\\&=3\left(x^2+8x+\left(\tfrac{8}{2}\right)^2\right)+50-3\left(\tfrac{8}{2}\right)^2\\&=3(x^2+8x+16)+50-48\\&=3(x+4)(x+4)+2\\&=3(x+4)^2+2\end{aligned}$$

Another way to put in vertex form. $\longrightarrow$ use vertex formula

First Vertex Formula: $x = \frac{-b}{2a}$, $y = f\left(\frac{-b}{2a}\right)$.

Lastly Vertex Form: $y = a(x-h)^2+k$

[Examples] Convert Standard to Vertex

① $y = x^2+8x+10$ $\quad a=1$, $b=-8$, $c=10$

$x = \frac{-b}{2a} = \frac{-(8)}{2(1)} = -4$

$y = f(-4) = (-4)^2+8(-4)+10 = -6$

$y = a(x-h)^2+k$
$= (x-(-4)^2)-6$
$= (x+4)^2-6$

② $y = x^2+6x+8$ $\quad a=1$, $b=6$, $c=8$

$x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$

$y = f(-3) = (-3)^2+6(-3)+8 = -1$

$y = a(x-h)^2+k$
$= (x-(-3))^2-1$
$= (x+3)^2-1$

③ $f(x) = x^2-4x+3$ $\quad a=1$, $b=-4$, $c=3$

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$y = f(2) = (2)^2-4(2)+3 = -1$

$y = a(x-h)^2+k$
$= (x-2)^2-1$

④ $y = 3x^2+24x+50$ $\quad a=3$, $b=24$, $c=50$

$x = -\frac{b}{2a} = \frac{-(24)}{2(3)} = -4$

$y = f(-4) = 3(-4)^2+24(-4)+50 = 2$

$y = a(x-h)^2+k$
$= 3(x-(-4))^2+2$
$= 3(x+4)^2+2$

$\boxed{2}$ Vertex to Standard $\longrightarrow$ Expand the equation.

- Expand the equation by multiplying & combining like terms.

[Examples] Convert Vertex to Standard.

① $y=(x-1)^2+8$
$= (x-1)(x-1)+8$
$= x^2-1x-1x+1+8$
$= x^2-2x+9$

② $y=-(x-4)^2+3$
$= -(x-4)(x-4)+3$
$= -(x^2-4x-4x+16)+3$
$= -(x^2-8x+16)+3$
$= -x^2+8x-16+3$
$= -x^2-8x-13$

③ $f(x)=2(x+3)^2-5$
$= 2(x+3)(x+3)-5$
$= 2(x^2+3x+3x+9)-5$
$= 2(x^2+6x+9)-5$
$= 2x^2+12x+18-5$
$= 2x^2+12x+13$

④ $f(x)=2(x+1)^2-2$
$= 2(x+1)(x+1)-2$
$= 2(x^2+1x+1x+1)-2$
$= 2(x^2+2x+1)-2$
$= 2x^2+4x+2-2$
$= 2x^2+4x$